Dodatna naloga 2

Določite največji poves nosilca in zasuk nosilca v členkastih podporah.

$F = 3\ \text{kN}$

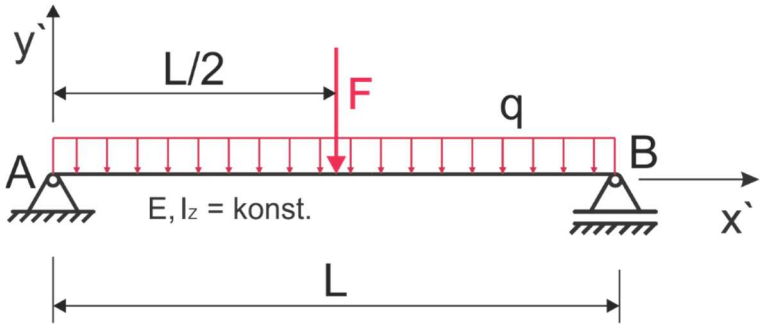
$q = 4\ \text{kN/m}$

$L = 1\ \text{m}$

$E = 200000\ \text{MPa}$

$I_z = 5 \cdot 10^5\ \text{mm}^4$

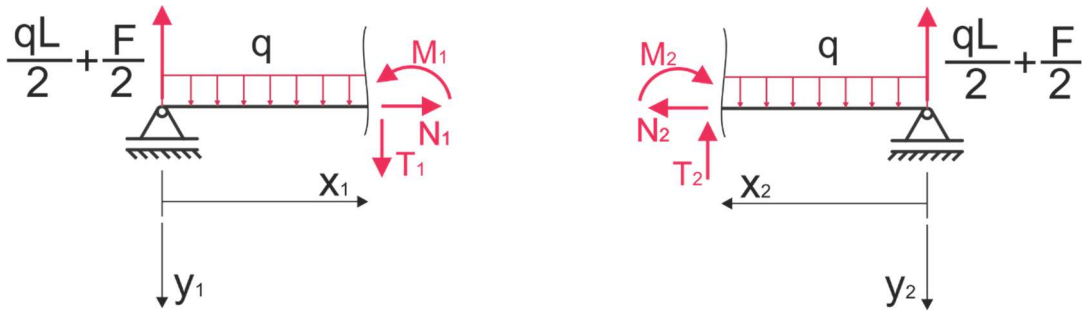
$y(x) = ?,\ x_{MAX} = ?,\ y_{MAX} = ?$



Rešitev:

- izračunamo reakcijske sile:

$$A_{x`} = 0,\ A_{y`} = \frac{qL}{2} + \frac{F}{2},\ B = \frac{qL}{2} + \frac{F}{2}$$



- izračunamo notranji upogibni moment:

$$M_1(x_1) = \frac{1}{2}(F + qL)x_1 - \frac{q{x_1}^2}{2},\quad M_2(x_2) = \frac{1}{2}(F + qL)x_2 - \frac{q{x_2}^2}{2}$$

- enačbo momenta vstavimo v diferencialno enačbo upogibnice in dvakrat integriramo. Dobimo:

$$y_1(x_1) = \frac{1}{2EI_z}\left(\frac{q{x_1}^4}{12} - (F + qL)\frac{{x_1}^3}{6}\right) + C_1x_1 + C_2$$

$$y_2(x_2) = \frac{1}{2EI_z}\left(\frac{q{x_2}^4}{12} - (F + qL)\frac{{x_2}^3}{6}\right) + C_3x_2 + C_4$$

- zapišemo robne pogoje in izračunamo vrednost integracijskih konstant:

1. $y_1(x_1 = 0) = 0$
2. $y_2(x_2 = 0) = 0,$
3. $y_1(x_1 = L/2) = y_2(x_2 = L/2)$
4. $y`_1(x_1 = L/2) = -y`_2(x_2 = L/2)$

$$C_2 = C_4 = 0,\ C_1 = C_3 = \frac{1}{EI_z}\left(\frac{qL^3}{24} + \frac{FL^2}{16}\right)$$

- poiščemo maksimalni poves nosilca (preverimo robove polj in stacionarne točke). Dobimo rezultat:

$$y_{\mathrm{MAX}} = y_1(x_1 = L/2) = y_2(x_2 = L/2) = \frac{5qL^4}{384EI_z} + \frac{FL^3}{48EI_z}$$

- izračunamo še zasuk nosilca v podporah:

$$\alpha \approx y`_1(x_1 = 0) = y`_2(x_2 = 0) = C_1 = \frac{17}{4800}\ \mathrm{rad}$$

Opomba: ker je problem simetričen glede na sredino nosilca, se da nalogo rešiti z zapisom ene same upogibnice z robnimi pogoji:

$$y_1(x_1 = 0) = 0, \quad y`_1(x_1 = L/2) = 0$$