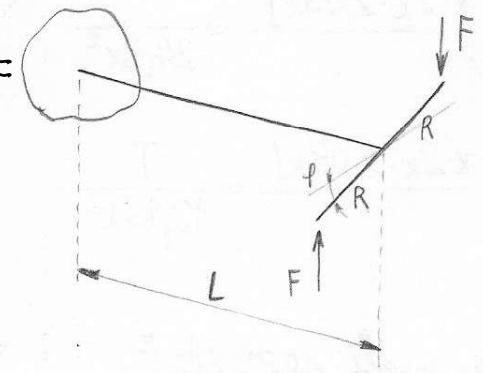
1.



$F = 2\,kN$ $\quad k = \frac{D}{d} = 1,1$

$R = 0,3\,m$

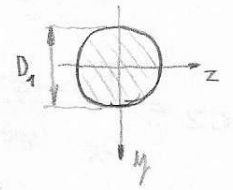
$L = 0,8\,m$

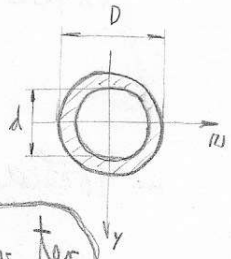
$\tau_{DOP} = 90\,MPa$

$\vartheta_{DOP} = 0,25\,°/m = 0,25 \cdot 10^{-3}\,°/mm$

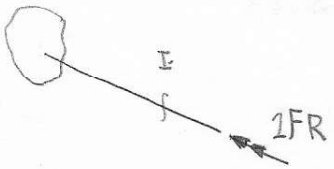
$G = 8,1 \cdot 10^4\,MPa$

$D_1, D, d = ?$

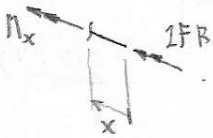




Dimenzionirajte gred in cev ter določite njuno razmerje tež.



I. polje:



$M_x = -2FR = M_T$

$0 \leq x \leq L$

$$\tau = \frac{M_T}{W_T} \leq \tau_{DOP}$$

$$\hat{\vartheta} = \hat{\nu} L = \frac{M_T L}{G J_T} \leq \hat{\nu}_{DOP} \cdot L$$

krožni prerez: $J_T = J_p = J_y + J_z$

$$J_T = \frac{\pi D_1^4}{64} \cdot 2 = \frac{\pi D_1^4}{32}$$

$$W_{T_{min}} = \frac{J_T \cdot 2}{D_1} = \frac{\pi D_1^3}{16}$$

} gred

$$J_T = \frac{\pi (D^4 - d^4)}{64} \cdot 2 = \frac{\pi (k^4 - 1) d^4}{32}$$

$$W_{T_{min}} = \frac{J_T \cdot 2}{D} = \frac{\pi (D^4 - d^4) \cdot 2}{32 \cdot D} = \frac{\pi \cdot (k^4 - 1) d^4}{16 \cdot k d} = \frac{\pi (k^4 - 1) d^3}{16 k}$$

} cev

dimenzioniranje gredi

$$\tau_{max} = \frac{M_T}{W_{T_{min}}} \leq \tau_{DOP}$$

$$\frac{-2FR \cdot 16}{\pi \cdot D_1^3} = -90$$

$$D_1 = \sqrt[3]{\frac{1FR \cdot 16}{\pi \cdot 90}} = 40,8\,mm$$

$$\vartheta = \frac{M_T}{G J_T} \frac{180}{\pi} \leq \vartheta_{DOP}$$

$$\frac{-2FR \cdot 180 \cdot 32}{G \cdot \pi \cdot D_1^4 \cdot \pi} = -0,25 \cdot 10^{-3}$$

$$D_1 = \sqrt[4]{\frac{1FR \cdot 180 \cdot 32}{G \cdot \pi^2 \cdot 0,25 \cdot 10^{-3}}} = 76,7\,mm$$

dimenzioniranje cevi

$$\tau_{max} = \frac{M_T}{W_{T_{min}}} \leq \tau_{DOP}$$

$$\frac{-1FR \cdot 16 \cdot k}{\pi \cdot (k^4 - 1) d^3} = -90$$

$$d = \sqrt[3]{\frac{1FR \cdot 16 \cdot k}{\pi \cdot (k^4 - 1) \cdot 90}} = 54,4\,mm \Rightarrow D = 59,84\,mm$$

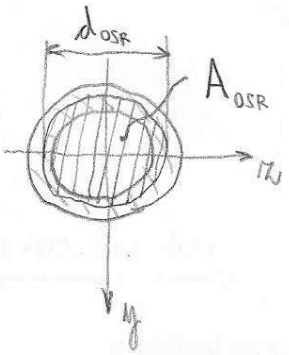
$$\vartheta = \frac{M_T}{G J_T} \frac{180}{\pi} \leq \vartheta_{DOP}$$

$$\frac{-2FR \cdot 32 \cdot 180}{G \cdot \pi \cdot (k^4 - 1) d^4 \pi} = -0,25 \cdot 10^{-3}$$

$$d = \sqrt[4]{\frac{2FR \cdot 32 \cdot 180}{G \cdot \pi^2 \cdot (k^4 - 1) \cdot 0,25 \cdot 10^{-3}}} = 92,91\,mm \Rightarrow D = 102,2\,mm$$

$$\frac{m_G}{m_C} = \frac{\rho \cdot \frac{\pi D_1^2}{4} \cdot L}{\rho \frac{\pi (D^2 - d^2)}{4} \cdot L} = \frac{D_1^2}{D^2 - d^2} = \frac{76,7^2}{102,2^2 - 92,91^2} = 3,2456$$

Ista naloga kot zgoraj, le da cev obravnavamo kot zaprt tankostenski prerez



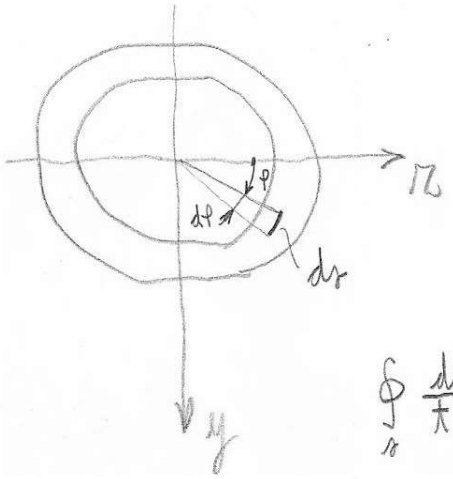
$$W_{T\,min} = 2 \cdot A_{OSR} \cdot t_{min}$$

$$J_T = \frac{4 \cdot A_{OSR}^2}{\oint_s \frac{dr}{t}}$$

$$A_{OSR} = \frac{\pi \cdot d_{OSR}^2}{4} = \frac{\pi \left(\frac{D+d}{2}\right)^2}{4}$$

$$A_{OSR} = \frac{\pi (k+1)^2 d^2}{16}$$

$$t_{min} = \frac{D-d}{2} = \frac{k-1}{2} d$$



$$\oint_s \frac{dr}{t} = \int_0^{2\pi} \frac{\pi_{OSR} \cdot d\varphi \cdot 2}{D-d} = \frac{2}{D-d} \int_0^{2\pi} \frac{D+d}{4} d\varphi = \frac{D+d}{D-d} \frac{1}{2} \int_0^{2\pi} d\varphi =$$

$$= \pi \frac{D+d}{D-d} = \pi \frac{k+1}{k-1}$$

$$W_{T\,min} = 2 \cdot \frac{\pi (k+1)^2 d^2}{16} \cdot \frac{k-1}{2} d = \frac{\pi (k+1)^2 (k-1)}{16} d^3$$

$$J_T = \frac{4 \cdot \pi^2 (k+1)^4 \cdot d^4 (k-1)}{256 \pi (k+1)} = \frac{\pi (k+1)^3 (k-1)}{64} d^4$$

$$\tau = \frac{M_T}{W_{T\,min}} \leq \tau_{DOP}$$

$$\frac{2FR \cdot 16}{\pi (k+1)^2 (k-1) d^3} = \tau_{DOP}$$

$$d = \sqrt[3]{\frac{32 \cdot F \cdot R}{\pi \cdot (k+1)^2 (k-1) \tau_{DOP}}}$$

$$d = 53{,}6 \text{ mm}$$

$$\vartheta = \frac{M_T}{G \cdot J_T} \cdot \frac{180}{\pi} \leq \vartheta_{DOP}$$

$$\frac{2FR \cdot 180 \cdot 64}{G \cdot \pi (k+1)^3 (k-1) d^4 \cdot \pi} = 0{,}25 \cdot 10^{-3}$$

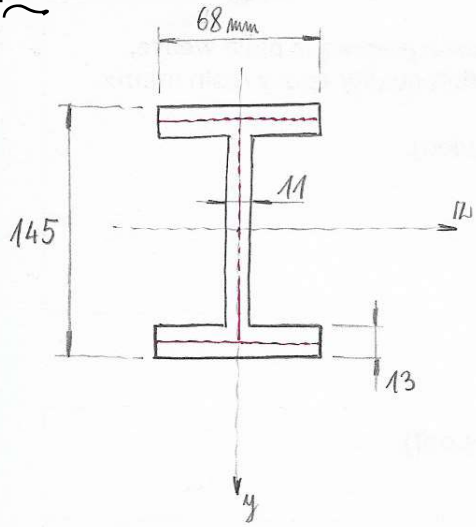
$$d = \sqrt[4]{\frac{2FR \cdot 180 \cdot 64}{G \cdot \pi^2 (k+1)^3 (k-1) \cdot 0{,}25 \cdot 10^{-3}}}$$

$$d = 92{,}96 \text{ mm}$$

$$d = 93 \text{ mm}$$

$$D = 101{,}3 \text{ mm}$$

2. Določite največjo strižno napetost v narisanem I preseku.



$M_T = 6{,}1\ kNm$

$\tau_{max} = ?$

$$J_T = \frac{n}{3} \cdot \sum_i t_i^3 \cdot h_i$$

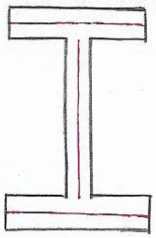
$$J_T = \frac{1{,}31}{3} \cdot (13^3 \cdot 68 \cdot 2 + 11^3 \cdot 132)$$

$$J_T = 207191{,}35\ mm^4$$

$$W_{T_{min}} = \frac{J_T}{t_{max}} = \frac{207191{,}35}{13} = 15937{,}8\ mm^3$$

$$\tau_{max} = \frac{M_T}{W_{T_{min}}} = \frac{6{,}1 \cdot 1000^2}{15937{,}8} = 382{,}74\ MPa$$

ali pa tako:



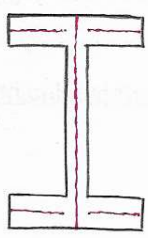
$$J_T = \frac{1{,}31}{3} \cdot (13^3 \cdot 68 \cdot 2 + 11^3 \cdot 119)$$

$$J_T = 199635{,}7\ mm^4$$

$$W_{T_{min}} = \frac{J_T}{t_{max}} = 15356{,}59\ mm^3$$

$$\tau_{max} = \frac{6{,}1 \cdot 1000^2}{15356{,}59} = 397{,}22\ MPa$$

ali pa tako:



$$J_T = \frac{1{,}31}{3} \cdot (13^3 \cdot 28{,}5 \cdot 4 + 11^3 \cdot 145)$$

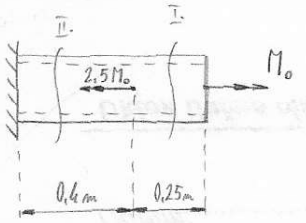
$$J_T = 193641{,}14\ mm^4$$

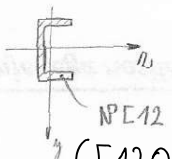
$$W_{T_{min}} = \frac{J_T}{t_{max}} = 14895{,}47\ mm^3$$

$$\tau_{max} = \frac{6{,}1 \cdot 1000^2}{14895{,}47} = 409{,}52\ MPa$$

Razlike so dokaj majhne, tako da so vsi trije načini v redu. Zadnji je najbolj "varen" pri dimenzioniranju.

3. ≈





(L120 profil)

$\nu_{DOP} = 0{,}25\ °/m$

$\tau_{DOP} = 90\ N/mm^2$

$G = 8{,}1 \cdot 10^4\ N/mm^2$

$M_0 = ?$

$2\pi \dots 360$

$x \dots 1°$

$x = \frac{2\pi}{360} = \frac{\pi}{180}\ rad$

$\varphi_{DOP} = \nu_{DOP} \cdot L = 0{,}25 \cdot 0{,}65 = 0{,}1625°$

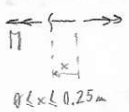
$\widehat{\varphi}_{DOP} = \frac{\pi}{180} \cdot 0{,}1625 = 2{,}8362 \cdot 10^{-3}$

$\widehat{\varphi} = \frac{M_T L}{G J_T}$

dim. na τ_{DOP}: $\tau_{max} = \frac{M_T}{W_T} \leq \tau_{DOP}$ ①

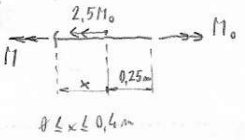
dim. na ν_{DOP}: $\nu = \frac{M_T}{G J_T} \leq \nu_{DOP}$ ②

① I. polje:



$M = M_0$

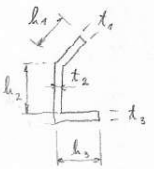
II. polje:



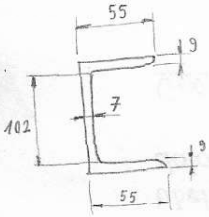
$M + 2{,}5 M_0 - M_0 = 0$

$M = -1{,}5 M_0$

$M_{T_{max}} = 1{,}5 M_0$



$W_T = \frac{\sum t_i^3 h_i}{3 t_{max}} \cdot \eta$



Približen izračun →

$W_T = \frac{9^3 \cdot 55 \cdot 2 + 7^3 \cdot 102}{3 \cdot 9} \cdot 1{,}12$

$W_T = 4777{,}67\ mm^3$

$\frac{1{,}5 M_0}{4777{,}67} = 90 \Rightarrow M_0 = 286660{,}3\ N_{mm} = 286{,}6\ N_m$

② $\widehat{\varphi}_{DOP} = \frac{M_0 \cdot 250}{8{,}1 \cdot 10^4 \cdot 42999{,}03} - \frac{1{,}5 M_0 \cdot 400}{8{,}1 \cdot 10^4 \cdot 42999{,}03} = 2{,}8362 \cdot 10^{-3}$

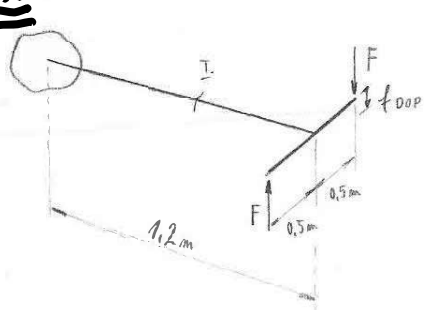
$M_0 (250 - 1{,}5 \cdot 400) = 8{,}1 \cdot 10^4 \cdot 42999{,}03 \cdot 2{,}8362 \cdot 10^{-3}$

$M_0 = \frac{8{,}1 \cdot 10^4 \cdot 42999{,}03 \cdot 2{,}8362 \cdot 10^{-3}}{1250 - 1{,}5 \cdot 4001} = 28223{,}6\ N_{mm} = 28{,}2\ N_m$

$J_T = \frac{\sum t_i^3 h_i}{3} \eta$

$J_T = \frac{9^3 \cdot 55 \cdot 2 + 7^3 \cdot 102}{3} \cdot 1{,}12 = 42999{,}03\ mm^4$

4.

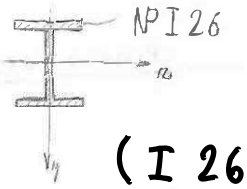


$f_{DOP} = 3\ mm$

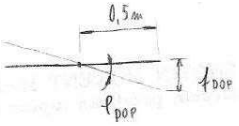
$\tau_{DOP} = 90\ N/mm^2$

$G = 8,1 \cdot 10^4\ N/mm^2$

$F = ?$



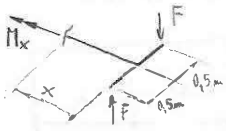
(I 260 profil)



$tg\ \varphi_{DOP} = \frac{f_{DOP}}{0,5}$

$\varphi_{DOP} = 0,3438°$

I. polje:



$M_x + F \cdot 0,5 + F \cdot 0,5 = 0$

$M_x = -F \cdot 1\ [Nm]$

$0 \leq x \leq 1,2\ m$

določimo F glede na τ_{DOP}:

$$\frac{M_T}{W_T} \leq \tau_{DOP}$$

$$W_T = \frac{\sum t_i^3 h_i}{3 t_{max}} \cdot 1,31$$

$$W_T = \frac{14,1^3 \cdot 113 \cdot 2 + 9,4^3 \cdot (260 - 2 \cdot 14,1)}{3 \cdot 14,1} \cdot 1,31$$

$W_T = 25582,39\ mm^3$

$$\frac{F \cdot 1000}{25582,39} = 90$$

$F = 2302,4\ N$

določimo F glede na φ_{DOP} oz. f_{DOP}:

$$\varphi = \frac{M_T L}{G J_T}$$

$$\varphi_{DOP} = \frac{180}{\pi} \cdot \frac{M_T L}{G J_T}$$

$$J_T = \frac{\sum t_i^3 h_i}{3} \cdot \eta$$

$$J_T = \frac{14,1^3 \cdot 113 \cdot 2 + 9,4^3 \cdot (260 - 2 \cdot 14,1)}{3} \cdot 1,31$$

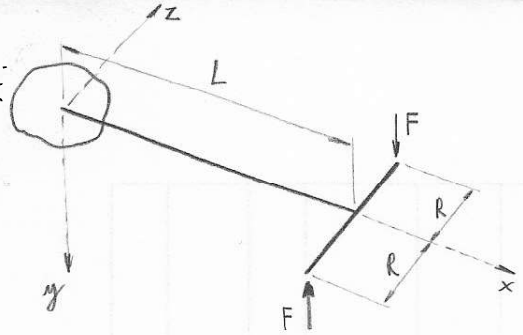
$J_T = 360711,69\ mm^4$

$$0,3438 = \frac{180}{\pi} \cdot \frac{F \cdot 1000 \cdot 1200}{8,1 \cdot 10^4 \cdot 360711,69}$$

$F = 146,1\ N$

$F_{DOP} = 146\ N$

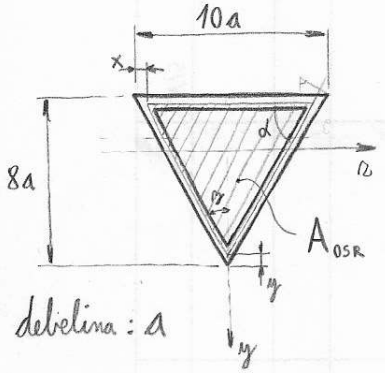
5.



$L = 1{,}5\ m$
$R = 0{,}3\ m$
$F = 200\ N$
$G = 8{,}1 \cdot 10^4\ MPa$
$f = 2$

$\vartheta_{DOP} = 0{,}25\ °/m$
$a = 5\ mm$

Izračunajte zasuk gredi. Ali je zasuk večji od dopustne vrednosti?



debelina: a

$$\vartheta = \frac{M_T L}{G \cdot J_T}$$

$$J_T = \frac{4 \cdot A_{OSR}^2}{\oint_r \frac{dr}{t}} = 2$$

$$A_{OSR} = (10a - 2x) \cdot (8a - \tfrac{a}{2} - y) \cdot \tfrac{1}{2}$$

$$A_{OSR} = (10a - \tfrac{a}{\sin\alpha}) \cdot (8a - \tfrac{a}{2} - \tfrac{a}{2\sin\gamma}) \tfrac{1}{2}$$

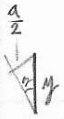
$$A_{OSR} = 28{,}917\, a^2$$

$$(A_{OSR} \approx (10a - a)(8a - a) \cdot \tfrac{1}{2} = 31{,}5\, a^3)$$

$$\operatorname{tg}\alpha = \frac{8a}{5a}$$
$$\alpha = 57{,}99°$$

$$\operatorname{tg}\gamma = \frac{5a}{8a}$$
$$\gamma = 32{,}01°$$

$$\sin\alpha = \frac{a/2}{x}$$
$$x = \frac{a}{2\sin\alpha}$$
$$x = \frac{a}{1{,}696}$$



$$\sin\gamma = \frac{a/2}{y}$$
$$y = \frac{a}{2\sin\gamma}$$
$$y = \frac{a}{1{,}06}$$

$$\oint_r \frac{dr}{t} = \frac{10a - 2x}{a} + \frac{2 \cdot 7{,}732\, a}{a} = 24{,}285$$

$$\oint_r \frac{dr}{t} = \frac{r_1}{a} + \frac{2r_2}{a}$$

$$\cos\gamma = \frac{8a - \frac{a}{2} - \frac{a}{2\sin\gamma}}{r_2}$$
$$r_2 = \frac{8a - \frac{a}{2} - \frac{a}{2\sin\gamma}}{\cos\gamma}$$
$$r_2 = 7{,}732\, a$$

$$J_T = \frac{4 \cdot (28{,}917\, a^2)^2}{24{,}285} = 137{,}73\, a^4$$

$$M_T = 2F \cdot R$$

$$f = \frac{180}{\pi} \frac{2FR \cdot L}{G \cdot 137{,}73\, a^4} = \frac{180}{\pi} \frac{2 \cdot 200 \cdot 300 \cdot 1500}{8{,}1 \cdot 10^4 \cdot 137{,}73 \cdot 5^4} = 1{,}479° > f_{DOP} = 0{,}375°$$

(Zasuk je večji od dopustnega)

$$\vartheta = \frac{1{,}479}{1{,}5\,m} = 0{,}986\ °/m$$

$$\vartheta_{DOP} = 0{,}25\ °/m \Rightarrow f_{DOP} = 0{,}25 \cdot 1{,}5 = 0{,}375°$$