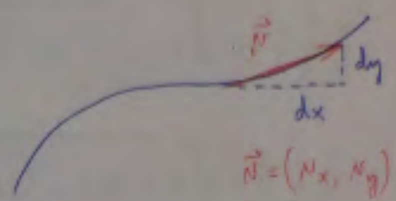
Naloga 3. Za podana hitrostna polja in začetne pogoje izpelji enačbe tokovnic.

Podatki: a.) $\vec{v}(x,y) = (-2y^2, -3x)$, $y(x=0) = 5$

b.) $\vec{v}(x,y) = (4x^2, -7y)$, $y(x=1) = 2$

c.) $\vec{v}(x,y) = (2-8y, 5e^{-x}-1)$, $y(x=0) = 4$

a.) $\vec{v}(x,y) = (-2y^2, -3x)$; $y(x=0) = 5$



$$\frac{dy}{dx} = \frac{v_y}{v_x} \rightarrow \frac{dy}{dx} = \frac{-3x}{-2y^2}$$

$$2y\,dy = 3x\,dx \rightarrow \int y^2 dy = \frac{3}{2}\int x\,dx$$

$$\frac{1}{3}y^3 = \frac{3}{2}\cdot\frac{x^2}{2} + C_0 \,/\cdot 3 \rightarrow y^3 = \frac{9}{4}x^2 + C \;/\sqrt[3]{\ } \quad \underline{y = \sqrt[3]{\frac{9}{4}x^2 + C}}$$

$$\hookrightarrow \sqrt[3]{\frac{9}{4}\cdot 0 + C} = 5 \rightarrow \sqrt[3]{C} = 5 \rightarrow C = 125 \longrightarrow \underline{y(x) = \sqrt[3]{\frac{9}{4}x^2 + 125}}$$

b.) $\vec{v}(x,y) = (4x^2, -7y)$; $y(x=1) = 2$ (spremenil zač. pogoj, da lahko dobimo definirano oz. določeno rešitev)

$$\frac{dy}{dx} = \frac{-7y}{4x^2} \longrightarrow \frac{1}{7}\int\frac{dy}{y} = -\frac{1}{4}\int\frac{dx}{x^2} \rightarrow \frac{1}{7}\ln y = -\frac{1}{4}\cdot\left(-\frac{1}{x}\right) + C_0 \,/\cdot 7$$

$$\ln y = \frac{7}{4}\frac{1}{x} + C_1 \rightarrow y = e^{\frac{7}{4}x^{-1}}\cdot C$$

$$\hookrightarrow e^{\frac{7}{4}\cdot\frac{1}{1}}\cdot C = 2 \rightarrow C = 2e^{-\frac{7}{4}} \longrightarrow \underline{y(x) = 2e^{\frac{7}{4}(x^{-1}-1)}}$$

c.) $\vec{v}(x,y) = \left(2-8y,\ 5e^{-x}-1\right)$; $y(x=0)=4$

$$\frac{dy}{dx} = \frac{5e^{-x}-1}{2-8y} \rightarrow \int (2-8y)\,dy = \int (5e^{-x}-1)\,dx$$

$$2\left(y-4\cdot\tfrac{1}{2}y^2\right) = -5e^{-x}-x+C$$

$$-4y^2+2y+5e^{-x}+x-C=0, \rightarrow -4y^2+2y+c(x)=0$$

$$y = \frac{-2\pm\sqrt{2^2-4(-4)c(x)}}{2(-4)} = \frac{1}{4}\left(1\pm\frac{1}{2}\sqrt{4+16c(x)}\right) = \underline{\frac{1}{4}\left(1\pm\sqrt{1+4c(x)}\right)}$$

I. Pozitivna veja:

$$y = \frac{1}{4}\left(1+\sqrt{1+4c(x)}\right) = \frac{1}{4}\left(1+\sqrt{1+4(5e^{-x}+x-C)}\right)$$

$$\hookrightarrow \frac{1}{4}\left(1+\sqrt{1+4(5\cdot 1+0-C)}\right) = 4 \ /\cdot 4$$

$$\sqrt{\underbrace{1+4(5-C)}} = 15 \ /^2 \rightarrow 1+4(5-C) = 225$$

$$\underline{C=-51}$$

$$\underline{y(x) = \frac{1}{4}\left(1+\sqrt{1+4(5e^{-x}+x+51)}\right)}$$

Privzamemo: $1+4(5-C)\geq 0$

II. Negativna veja:

$$y = \frac{1}{4}\left(1-\sqrt{1+4c(x)}\right) = \frac{1}{4}\left(1-\sqrt{1+4(5e^{-x}+x-C)}\right) \rightarrow \underline{y(x) = \frac{1}{4}\left(1-\sqrt{1+4(5e^{-x}+x+51)}\right)}$$

$$\hookrightarrow \frac{1}{4}\left(1-\sqrt{1+4(5\cdot 1+0-C)}\right) = 4 \ /\cdot 4$$

$$\sqrt{1+4(5-C)} = -15 \ /^2$$

$\vdots$

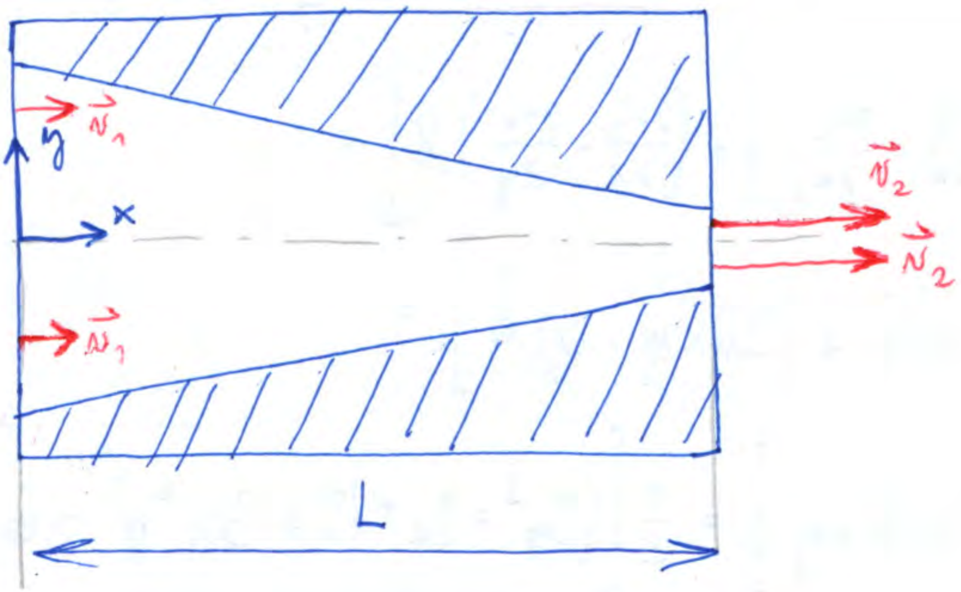
$$C=-51$$

--> Tokovnico, ki dejansko ustreza podanemu robnemu pogoju predstavlja rešitev pozitivne veje.

Naloga 4. Podano je hitrostno polje za enodimenzionalni stacionarni tok skozi konvergentno šobo, prikazano na sliki. Določi pospešek hitrostnega polja (v Eulerjevem popisu gibanja) in položaj ter pospešek fluidnega delca ob času t (Ob času t = 0 s se delec nahaja na vstopu v šobo - x = 0 m). Hitrost je podana v m/s.

Podatki: $v(x) = v_0\left(1+\frac{x}{L}\right)$, $v_1 = v(x=0) = v_0$, $v_2 = v(x=L) = 2v_0$

$a = ?$ $x_D(t) = ?$ $a_D(t) = ?$



→ Pospešek / vektorsko polje pospeška:

$$\vec{v}(x) = \left(v_0\left(1+\frac{x}{L}\right), 0\right) = v_0\left(1+\frac{x}{L}\right)\vec{i}, \quad \vec{a} = \frac{\partial \vec{v}}{\partial t} + v_x\frac{\partial \vec{v}}{\partial x} + v_y\frac{\partial \vec{v}}{\partial y} + v_z\frac{\partial \vec{v}}{\partial z}$$

↳ Ker imamo opravka z 1D (enodimenzionalnim) stacionarnim tokom, sledi:

$$\frac{\partial \vec{v}}{\partial t} = 0, \quad v_y = 0, \quad v_z = 0, \quad \frac{\partial \vec{v}}{\partial y} = 0, \quad \frac{\partial \vec{v}}{\partial z} = 0$$

$$\vec{a} = v_x\frac{\partial \vec{v}}{\partial x} = v_0\left(1+\frac{x}{L}\right)\cdot\frac{\partial}{\partial x}\left(v_0\left(1+\frac{x}{L}\right), 0\right) = v_0\left(1+\frac{x}{L}\right)\cdot\left(\frac{v_0}{L}, 0\right) =$$

$$= \frac{v_0^2}{L}\left(1+\frac{x}{L}\right)\cdot(1, 0) = \frac{v_0^2}{L}\left(1+\frac{x}{L}\right)\vec{i} \quad \left[\frac{m}{s^2}\right]$$

→ Pospešek / vektorsko polje pospeška:

$$\vec{v}(x) = \left(v_0\left(1+\frac{x}{L}\right), 0\right) = v_0\left(1+\frac{x}{L}\right)\vec{i}\ , \quad \vec{a} = \frac{\partial \vec{v}}{\partial t} + v_x\frac{\partial \vec{v}}{\partial x} + v_y\frac{\partial \vec{v}}{\partial y} + v_z\frac{\partial \vec{v}}{\partial z}$$

↳ Ker imamo opravka z 1D (enodimenzionalnim) stacionarnim tokom, sledi:

$$\frac{\partial \vec{v}}{\partial t} = 0, \quad v_y = 0, \quad v_z = 0, \quad \frac{\partial \vec{v}}{\partial y} = 0, \quad \frac{\partial \vec{v}}{\partial z} = 0$$

$$\vec{a} = v_x\frac{\partial \vec{v}}{\partial x} = v_0\left(1+\frac{x}{L}\right)\cdot\frac{\partial}{\partial x}\left(v_0\left(1+\frac{x}{L}\right), 0\right) = v_0\left(1+\frac{x}{L}\right)\cdot\left(\frac{v_0}{L}, 0\right) =$$

$$= \underline{\frac{v_0^2}{L}\left(1+\frac{x}{L}\right)\cdot(1, 0) = \frac{v_0^2}{L}\left(1+\frac{x}{L}\right)\vec{i}} \ \left[\frac{m}{s^2}\right]$$

→ Položaj fluidnega delca ob času t - določimo <u>tirnico</u> delca: $\bar{x}$ ... koordinata položaja fluidnega delca

$$\frac{d\bar{x}}{dt} = v_x \rightarrow dt = \frac{d\bar{x}}{v_x(\bar{x})} \rightarrow \int_{t_0}^{t} dt = \int_{x_0}^{x}\frac{d\bar{x}}{v_x(\bar{x})}$$

$$1+\frac{\bar{x}}{L} = r \quad dr = \frac{d\bar{x}}{L} \rightarrow dx = L\,dr$$

$$t - t_0 = \int_{x_0}^{x}\frac{d\bar{x}}{v_0\left(1+\frac{\bar{x}}{L}\right)} = \frac{1}{v_0}\cdot L\int_{1+\frac{\bar{x}_0}{L}}^{1+\frac{\bar{x}}{L}}\frac{dr}{r} = \frac{L}{v_0}\int_{1+\frac{\bar{x}_0}{L}}^{1+\frac{\bar{x}}{L}}\frac{dr}{r} = \frac{L}{v_0}\ln(r)\Big|_{1+\frac{\bar{x}_0}{L}}^{1+\frac{\bar{x}}{L}} =$$

$$= \frac{L}{v_0}\ln\left(\frac{1+\frac{\bar{x}}{L}}{1+\frac{\bar{x}_0}{L}}\right); \quad t_0 = 0,\ \bar{x}_0 = 0 \rightarrow t = \frac{L}{v_0}\ln\left(1+\frac{\bar{x}}{L}\right) \ / \exp$$

$$1+\frac{\bar{x}}{L} = e^{\frac{v_0}{L}t} \longrightarrow \underline{\bar{x}(t) = L\left(e^{\frac{v_0}{L}t} - 1\right)} \ [m]$$

→ Pospešek fluidnega delca - drugi odvod funkcije tirnice po času:

$$\bar{a}_x(t) = \frac{d^2}{dt^2}\bar{x}(t) = \frac{d^2}{dt^2}L\left(e^{\frac{v_0}{L}t} - 1\right) = L\,e^{\frac{v_0}{L}t}\cdot\frac{v_0^2}{L^2} = \underline{\frac{v_0^2}{L}e^{\frac{v_0}{L}t}} \ \left[\frac{m}{s^2}\right]$$