

Naloga 1. Za podano hitrostno polje ugotovi, ali je gostota fluida konstantna? V podani točki prostora tudi določi kotno hitrost in pospešek fluida.

Podatki: $\vec{v}(x, y, z) = (0, ay + bz, cz)$; $a = 6.5$, $b = 3.0$, $c = 7.2$, $T = (1, 2, 5)$

Ali je ρ konstanta? $\omega(x_T, y_T, z_T) = ?$, $\vec{a}(x_T, y_T, z_T) = ?$

→ Gostota hitrostnega polja: $\left[\frac{m}{s}\right]$

Dodatk: $\vec{v}(x, y, z) = (0, ay + bz, cyz)$; $a = 6.5$, $b = 3.0$, $c = 7.2$, $T = (1, 2, 5)$

Ali je ρ konstanta? $\omega(x_T, y_T, z_T) = ?$, $\vec{a}(x_T, y_T, z_T) = ?$

→ Enote hitrostnega polja: $\left[\frac{m}{s}\right]$

a.) Preizkus / test strujnosti: → kont. enačba: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \vec{\nabla} \rho + \rho \vec{\nabla} \cdot \vec{v} = 0 \rightarrow \text{če } \rho = \text{konst.}: \frac{\partial \rho}{\partial t} = 0, \vec{\nabla} \rho = 0$$

tedi pogoj $\rho \vec{\nabla} \cdot \vec{v} = 0 \rightarrow \vec{\nabla} \cdot \vec{v} = 0$

$$\hookrightarrow \vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0 + \frac{\partial}{\partial y}(ay + bz) + \frac{\partial}{\partial z}(cyz) = a + cy = 6.5 + 7.2y \neq 0$$

Ker $\vec{\nabla} \cdot \vec{v} \neq 0$, tedi da tudi ρ ni konst. $\vec{i}, \vec{j}, \vec{k} \dots$ enotski vektorji (enakovredno $\hat{e}_x, \hat{e}_y, \hat{e}_z$)

$$\begin{aligned} b.) \vec{\omega} &= \frac{1}{2} \vec{\nabla} \times \vec{v} = \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \frac{1}{2} \left[\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) \vec{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) \vec{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) \vec{k} \right] = \\ &= \frac{1}{2} \left[\left(\frac{\partial}{\partial y}(cyz) - \frac{\partial}{\partial z}(ay + bz) \right) \vec{i} + \left(\frac{\partial}{\partial z}(0) - \frac{\partial}{\partial x}(cyz) \right) \vec{j} + \left(\frac{\partial}{\partial x}(ay + bz) - \frac{\partial}{\partial y}(0) \right) \vec{k} \right] = \\ &= \frac{1}{2} \left[(cz - b) \vec{i} + 0 \vec{j} + 0 \vec{k} \right] = \frac{1}{2} (cz - b) \vec{i} \end{aligned}$$

$$\hookrightarrow \text{v točki } T: \vec{\omega}(x_T, y_T, z_T) = \vec{\omega}(1, 2, 5) = \frac{1}{2} (7.2 \cdot 5 - 3.0) \vec{i} = 16.5 \frac{\text{rad}}{\text{s}} \vec{i}$$

c.) $\vec{a} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} = \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} + v_z \frac{\partial \vec{v}}{\partial z} =$

$$= \frac{\partial}{\partial t} (v_x, v_y, v_z) + v_x \frac{\partial}{\partial x} (v_x, v_y, v_z) + v_y \frac{\partial}{\partial y} (v_x, v_y, v_z) + v_z \frac{\partial}{\partial z} (v_x, v_y, v_z) =$$

$$= (0, 0, 0) + 0 \cdot (0, 0, 0) + (ay + bz) \cdot (0, a, cy) + cyz \cdot (0, b, cy) =$$

$$\begin{aligned} \hookrightarrow \text{v točki } T: \vec{a}(x_T, y_T, z_T) &= \vec{a}(1, 2, 5) = (6.5 \cdot 2 + 3.0 \cdot 5) \cdot (0, 6.5, 7.2 \cdot 5) + 7.2 \cdot 2 \cdot 5 \cdot (0, 3.0, 7.2 \cdot 2) = \\ &= (28 \cdot (0, 6.5, 36) + 72 \cdot (0, 3.0, 14.4)) = (0, 398, 2044.8) \frac{m}{s^2} \end{aligned}$$

Naloga 2. Za podano hitrostno polje določi vrednosti konstant a in b , da bo tok nestisljiv oziroma gostota konstantna. Za podano točko prostora in časovni trenutek tudi določi kotno hitrost in pospešek.

Podatki: $\vec{v}(x, y, z) = (x^3 + 2z^2, ay^2t + bx^2y, + 7, y^3 - 2yzt)$, $T_A = (-1, 2, \frac{1}{2})$, $t = 3s$

$a = ?$, $b = ?$, $\vec{\omega}(x_T, y_T, z_T) = ?$, $\vec{a}(x_T, y_T, z_T) = ?$

Podatki: $\vec{n}(x, y, z) = (x^3 + 2z^2, ay^2t + bx^2y + 7, y^3 - 2yzt)$, $T_A = (-1, 2, \frac{1}{2})$, $t = 3n$

$a = ?$, $b = ?$, $\vec{\omega}(x_T, y_T, z_T) = ?$, $\vec{a}(x_T, y_T, z_T) = ?$

a.) $\hookrightarrow$ Pogoj nerazločnosti: $\vec{\nabla} \cdot \vec{n} = 0 \rightarrow \vec{\nabla} \cdot \vec{n} = \frac{\partial N_x}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{\partial N_z}{\partial z} =$

$$= 3x^2 + 2ayt + bx^2 - 2yt = 0$$

Da mora veljati pogoj $\vec{\nabla} \cdot \vec{n} = 0$ v celotnem prostoru $(x, y, z \in \mathbb{R})$ in ob vseh t ($t \in \mathbb{R}$),

leda: $(3+b)x^2 + (a-1)2yt = 0 \rightarrow 3+b=0, a-1=0 \rightarrow$ $a=1$
 $b=-3$

$\hookrightarrow \vec{n}(x, y, z, t) = (x^3 + 2z^2, y^2t - 3x^2y + 7, y^3 - 2yzt) \left[\frac{m}{n} \right]$

b.) $\omega = \frac{1}{2} \vec{\nabla} \times \vec{n} = \frac{1}{2} \left[\left(\frac{\partial N_z}{\partial y} - \frac{\partial N_y}{\partial z} \right) \vec{i} + \left(\frac{\partial N_x}{\partial z} - \frac{\partial N_z}{\partial x} \right) \vec{j} + \left(\frac{\partial N_y}{\partial x} - \frac{\partial N_x}{\partial y} \right) \vec{k} \right] =$

$$= \frac{1}{2} \left[(3y^2 - 2zt) \vec{i} + (3x^2 - 0) \vec{j} + (2bx^2y - 0) \vec{k} \right] =$$

$$= \frac{1}{2} \left[(3y^2 - 2zt) \vec{i} + 3x^2 \vec{j} + 2bx^2y \vec{k} \right] = \frac{1}{2} \left[(3y^2 - 2zt) \vec{i} + 3x^2 \vec{j} - 6x^2y \vec{k} \right]$$

V točki T: $\vec{\omega}(x_T, y_T, z_T, t=3n) = \frac{1}{2} \left[(3 \cdot 2^2 - 2 \cdot \frac{1}{2} \cdot 3) \vec{i} + 3(-1)^2 \vec{j} - 6 \cdot (-1) \cdot 2 \vec{k} \right] =$
 $(4.5, 1.5, 6.0) \frac{m}{n}$

c.) $\vec{a} = \frac{\partial \vec{n}}{\partial t} + \vec{u}_x \frac{\partial \vec{n}}{\partial x} + \vec{u}_y \frac{\partial \vec{n}}{\partial y} + \vec{u}_z \frac{\partial \vec{n}}{\partial z} =$

$$= (0, y^2, -2yz) + (x^3 + 2z^2) \cdot (3x^2, -6xy, 0) + (y^2t - 3x^2y + 7) \cdot (0, 2yt - 3x^2, 3y^2 - 2zt) +$$

$$+ (y^3 - 2yzt) \cdot (4z, 0, -2yt)$$

V točki T: $\vec{a}(x_T, y_T, z_T, t=3n) = (0, 2^2, -2 \cdot 2 \cdot \frac{1}{2}) + ((-1)^3 + 2 \cdot (\frac{1}{2})^2) \cdot (3(-1)^2, -6 \cdot (-1) \cdot 2, 0) +$
 $(2^2 \cdot 3 - 3 \cdot (-1)^2 \cdot 2 + 7) \cdot (0, 2 \cdot 2 \cdot 3 - 3 \cdot (-1)^2, 3 \cdot 2^2 - 2 \cdot \frac{1}{2} \cdot 3) +$
 $(2^3 - 2 \cdot 2 \cdot \frac{1}{2} \cdot 3) \cdot (4 \cdot \frac{1}{2}, 0, -2 \cdot 2 \cdot 3) =$ $(12.5, 115, 91) \frac{m}{n^2}$

Naloga 3. Za podana hitrostna polja in začetne pogoje izpelji enačbe tokovnic.

Podatki: a.) $\vec{v}(x, y) = (-2y^2, -3x)$, $x(0) = 5$, $y(x=0) = 5$

b.) $\vec{v}(x, y) = (4x^2, -7y)$, $y(x=0) = 2$

c.) $\vec{v}(x, y) = (2 - 8y, 5e^{-x} - 1)$, $y(x=0) = 4$

Naloga 4. Podano je hitrostno polje za enodimenzionalni stacionarni tok skozi konvergentno šobo, prikazano na sliki. Določi pospešek hitrostnega polja (v Eulerjevem popisu gibanja) in položaj ter pospešek fluidnega delca ob času t (Ob času $t = 0$ s se delec nahaja na vstopu v šobo - $x = 0$ m). Hitrost je podana v m/s.

Podatki: $v(x) = v_0 \left(1 + \frac{x}{L}\right)$, $v_1 = v(x=0) = v_0$, $v_2 = v(x=L) = 2v_0$

$a = ?$, $x_0(t) = ?$, $a_0(t) = ?$

